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STABILITY OF A NEW POLYNOMIAL DISCRETE RECURRENT NEURAL NETWORK

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Abstract. In this paper we construct a new polynomial discrete recurrent neural network, from the fixed points of quadratic and cubic polynomials. Our goal is to establish a method to construct the matrix of the synaptic weights of the network, and guarantee the stability of several fixed points attractors given previously. In addition, we give an application to recognition of four patterns.

Keywords: *discrete neural network, recurrent neural network, stability, fixed point, Hopfield network.*

AMS Subject Classification: *37D05, 37D25, 37D40, 37D45.*

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1. Introduction

An artificial neural network is a mathematical model based on the nervous system of living beings. An important property of these models is the ability to acquire and store information. At present there are a variety of continuous and discrete neural network models.

In 1982 Jhon Hopfield [2] presented a new model of a recurrent discrete neural network, which constitutes an associative memory with many applications such as recognition of patterns, images, signals, etc.

In this paper we construct a new recurrent polynomial discrete neural network, in the sense that we use fixed points of quadratic and cubic polynomial functions, constructed by Rubio and Hernández (2015). This new neural network generalizes the recurrent quadratic neural network [8], which contained a single previously fixed point attractor.

Our goal is to establish a method to construct the matrix of synaptic weights, which will guarantee the stability of several fixed points attractors given previously. In addition, we give an application of our network in the area of pattern recognition.

2. Quadratic and cubic functions

In this section, we present some results obtained by Rubio and Hernandez [6]. In the quadratic case, two points are given as a priori fixed points

$x_0, x_1 \in \mathbb{R}$, $x_0 < x_1$, and we determine the quadratic function:

$$f(x) = Ax^2 + Bx + C, \tag{1}$$

where:

$$\begin{cases} A = \frac{y_m - x_m}{(x_m - x_1)(x_m - x_0)} \\ B = \frac{y_m(x_0 + x_1) - x_0 x_1 - x_m^2}{(x_1 - x_m)(x_m - x_0)} \\ C = \frac{x_0 x_1 (y_m - x_m)}{(x_m - x_1)(x_m - x_0)}. \end{cases} \tag{2}$$

The point (x_m, y_m) is given, such that $(x_0, x_0), (x_1, x_1)$ and (x_m, x_m) are not collinear.

By using theorem (5.1) of [6], with $x_m = x_0 - \varepsilon$, $y_m = x_0$, $\varepsilon = 0.1$, we obtain:

a) x_0 is an attractor fixed point, (3)

b) x_1 is a repellent fixed point.

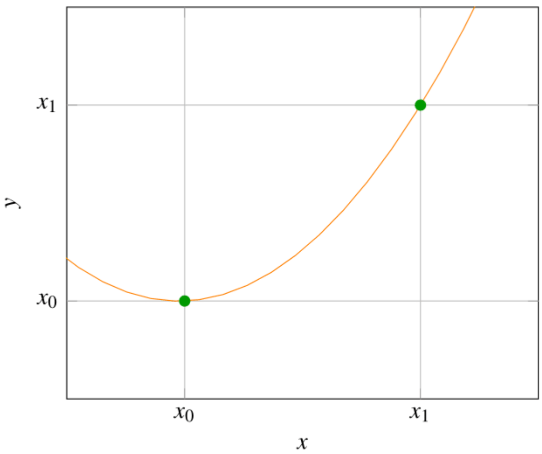


Fig.1. x_0 is an attractor fixed point.

By using theorem (5.4) of [6], with $x_m = x_1 + \varepsilon$, $y_m = x_1$, $\varepsilon = 0.1$, we obtain:

a) x_0 is a repellent fixed point. (4)

b) x_1 is an attractor fixed point.

c) In the cubic case, the points$x_0, x_1, x_2 \in \mathbb{R}$, $x_0 < x_1 < x_2$, are given as fixed points given previously, and we determine the cubic function:

d) $f(x) = Ax^3 + Bx^2 + Cx + D$ (5)

e) where, by theorem (3.1) of [6], we obtain:

$$\begin{cases} A = \frac{-(y_m - x_m)}{(x_0 - x_m)(x_1 - x_m)(x_2 - x_m)} \\ B = \frac{-(x_m - y_m)(x_0 + x_1 + x_2)}{(x_0 - x_m)(x_m - x_1)(x_m - x_2)} \\ C = \frac{-x_0 x_1 x_2 + x_0 x_1 y_m + x_0 x_2 y_m - x_0 x_m^2 + x_1 x_2 y_m - x_1 x_m^2 - x_2 x_m^2 + x_m^3}{(x_0 - x_m)(x_m - x_1)(x_m - x_2)} \\ D = \frac{-x_0 x_1 x_2 (x_m - y_m)}{(x_0 - x_m)(x_m - x_1)(x_m - x_2)} \end{cases} \tag{6}$$

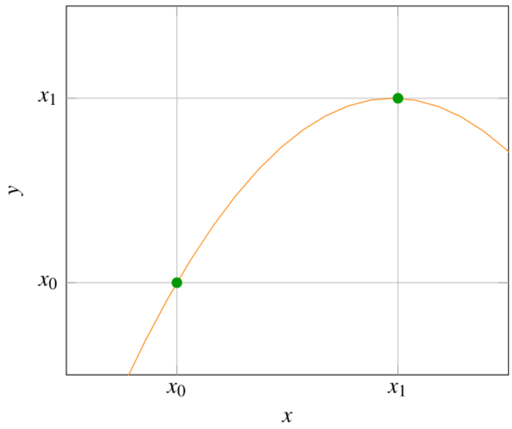


Fig.2. x_1 is an attractor fixed point

The point (x_m, y_m) is given, such that
(x_0, x_0), (x_1, x_1), (x_2, x_2) and (x_m, x_m) are no collinear.
If $x_m = x_2 + \varepsilon$, $y_m = x_2$, $\varepsilon = 0.1$, we obtain:

a) x_0 and x_2 are attractor fixed points, (7)

b) x_1 is a repellent fixed point.

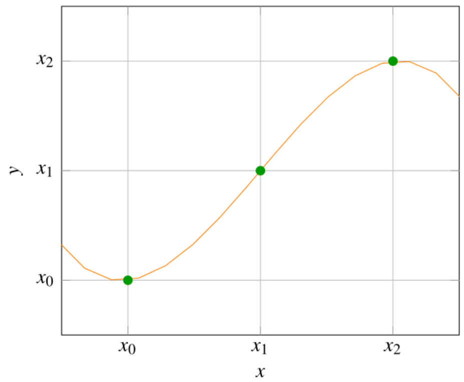


Fig.3. x_0 and x_2 are attractor fixed points

3. Generalization

In this section, we extend the theory about recurrent quadratic neural network [8], in the case of several attractor fixed points.

We consider $\mathbb{Z}^n$, where $\mathbb{Z}$ is the set of integer numbers, and let $z_0 \in \mathbb{Z}^n$ be, $z_0 = (z_0^1, \dots, z_0^n)$ such that:

$$z_0 = (\underbrace{0, \dots, 0}_{N_0}, z_0^{N_0+1}, \dots, z_0^n), \tag{8}$$

where $1 \le N_0 \le n-2$.

Following to Rubio and Hernández [7], by using z_0, we obtain the matrix W_0, making:

$$1°.\quad w_{i,N_0+1} z_0^{N_0+1} + \dots + w_{i,n} z_0^n = 0, \quad \forall i = 1, \dots, N_0. \tag{9}$$

Without loss of generality, we solve for the last term:

$$\sum_{j=N_0+1}^{n-1} w_{i,j} z_0^j + w_{i,n} z_0^n = 0,$$

$$w_{i.n} = -\sum_{j=N_0+1}^{n-1} w_{i.j}\,\frac{z_0^j}{z_0^n}. \tag{10}$$

$2°$. $\sum_{\substack{j=N_0+1\\ j\neq i}}^{n} w_{i,j}\, z_0^j + w_{i,i} z_0^i = z_0^i\,,\ \forall\, i = N_0+1, \dots, n.$ (11)

Then:

$$w_{i.i} = 1 - \sum_{\substack{j=N_0+1\\ j\neq i}}^{n} w_{i,j}\,\frac{z_0^j}{z_0^i}. \tag{12}$$

Therefore, the matrix W_0 have the form:

$$W_0 = \begin{pmatrix} W^1 & W^2 \\ W^3 & W^4 \end{pmatrix}, \tag{13}$$

where:

$$W^1 = \begin{pmatrix} w_{1,1} & \cdots & w_{1,N_0} \\ \vdots & \ddots & \vdots \\ w_{N_0,1} & \cdots & w_{N_0,N_0} \end{pmatrix}, \quad W^2 = \begin{pmatrix} w_{1,N_0+1} & \cdots & w_{1,n} \\ \vdots & \ddots & \vdots \\ w_{N_0,N_0+1} & \cdots & w_{N_0,n} \end{pmatrix}$$

$$W^3 = \begin{pmatrix} w_{N_0+1,1} & \cdots & w_{N_0+1,N_0} \\ \vdots & \ddots & \vdots \\ w_{n,1} & \cdots & w_{n,N_0} \end{pmatrix}, \quad W^4 = \begin{pmatrix} w_{N_0+1,N_0+1} & \cdots & w_{N_0+1,n} \\ \vdots & \ddots & \vdots \\ w_{n,N_0+1} & \cdots & w_{n,n} \end{pmatrix}.$$

We note that $w_{i,n}$, $\forall\, i = 1, \dots, N_0$, are obtained by using (10), and $w_{i,i}$,$\forall\, i = N_0 + 1, \dots, n$, are obtained by using (12).

Now, we state and show some results, for that z_0 will denote both the vector in $\mathbb{Z}^n$ and the matrix of order $n \times 1$, whose elements are the same of z_0.

Theorem 1. Let W_0 be the matrix (13). Then:

$$W_0 . z_0 = z_0. \tag{14}$$

Proof. By (8):

$$z_0 = \Big(\underbrace{0, \dots, 0}_{N_0}, z_0^{N_0+1}, \dots, z_0^n\Big).$$

We obtain:

a) $\sum_{j=1}^{n} w_{i,j}\, z_0^j = \sum_{j=N_0+1}^{n} w_{i,j}\, z_0^j = \sum_{j=N_0+1}^{n-1} w_{i,j}\, z_0^j + w_{i,n} z_0^n =$

$= \sum_{j=N_0+1}^{n-1} w_{i,j}\, z_0^j + z_0^n \left(-\sum_{j=N_0+1}^{n-1} w_{i.j}\,\frac{z_0^j}{z_0^n}\right) = 0,\ \forall i = 1, \dots, N_0,$ by (10).

b) $\sum_{j=1}^{n} w_{i,j}\, z_0^j = \sum_{j=N_0+1}^{n} w_{i,j}\, z_0^j\,,\ \forall i = N_0 + 1, \dots, n$

$$= z_0^i w_{i,i} + \sum_{\substack{j=N_0+1\\ j\neq i}}^{n} w_{i,j}\, z_0^j$$

$$= z_0^i\Big(1 - \sum_{\substack{j=N_0+1\\ j\neq i}}^{n} w_{i.j}\,\frac{z_0^j}{z_0^i}\Big) + \sum_{\substack{j=N_0+1\\ j\neq i}}^{n} w_{i,j}\, z_0^j,$$

$$= z_0^i\ .$$

by (12).

Since (a) and (b), we obtain that $W_0 . z_0 = z_0$.

The next, we propose a methodology to achieve our goal.

So, we consider the Hamming space $H^n = \{-1,\ 1\}^n$, and let:

$$x_{p_1}, x_{p_2}, \dots, x_{p_r} \in H^n = \{-1,\ 1\}^n\ ,\ x_{p_i} \neq x_{p_j},\ \ i \neq j \tag{15}$$

Our method consist of the following steps:

1. Let $z_r = \sum_{j=1}^{r} x_{p_j}$ be. Then: $z_r^i = \sum_{j=1}^{r} x_{p_j}^i$, $\forall i = 1, \dots, n$.
2. Now, we construct the vector $z_0 \in \mathbb{Z}^n$, taking into account that:
   a) If $z_r^i \neq \pm r$, then $z_0^i = 0$. (16)
   b) If $z_r^i = \pm r$, then $z_0^i = z_r^i$.
3. Let N_0 be the number of zeros of z_0. It must be fulfilled that:
$$1 \le N_0 \le n-2. \tag{17}$$
4. Without loss of generality, we suppose that:
$$z_0 = \left(\underbrace{0,\dots,0}_{N_0}, z_0^{N_0+1}, \dots, z_0^n\right). \tag{18}$$
5. By using the vector z_0 given by (8), we obtain the matrix W_0 given by (13). Moreover, by theorem 1, we obtain that $W_0 . z_0 = z_0$.
6. Since the matrix W_0 have N_0 free columns, we will assign values to its components, such a way we can obtain the matrix W, according to the following rule:
   a) $w_{i,i} = 1,\ \ \forall\ i = 1, \dots, N_0$. (19)
   b) $w_{i,i} = 0,\ \ \forall\ i = 1, \dots, n\ ,\ \ \forall\ j = 1, \dots, N_0\ ,\ \ i \neq j$. (20)

Therefore, the matrix W is:

$$W = \begin{pmatrix} W^1 & W^2 \\ W^3 & W^4 \end{pmatrix}, \tag{21}$$

where:

$$W^1 = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \quad , \quad W^2 = \begin{pmatrix} w_{1,N_0+1} & \cdots & w_{1,n} \\ \vdots & \ddots & \vdots \\ w_{N_0,N_0+1} & \cdots & w_{N_0,n} \end{pmatrix},$$

$$W^3 = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \quad , \quad W^4 = \begin{pmatrix} w_{N_0+1,N_0+1} & \cdots & w_{N_0+1,n} \\ \vdots & \ddots & \vdots \\ w_{n,N_0+1} & \cdots & w_{n,n} \end{pmatrix}.$$

Theorem 2. Let W be the matrix given by (21), and x_{p_j} ,$\forall\, j = 1, \dots, r$ the vectors given by (15). Then:

$$W x_{p_j} = x_{p_j}\ \ ,\ \ \forall\, j = 1, \dots, r. \tag{22}$$

Proof. Let $x_{p_j} = \left(x_{p_j}^1, x_{p_j}^2, \dots, x_{p_j}^n\right)$, $\forall\, j = 1, \dots, r$. We obtain:

a) $\sum_{k=1}^{n} w_{i,k}\, x_{p_j}^k = \sum_{k=1}^{N_0} w_{i,k}\, x_{p_j}^k + \sum_{k=N_0+1}^{n} w_{i,k}\, x_{p_j}^k$

$$= x_{p_j}^i + \sum_{k=N_0+1}^{n-1} w_{i,k}\, x_{p_j}^k + x_{p_j}^n w_{i,n}$$

$= x_{p_j}^i + \sum_{k=N_0+1}^{n-1} w_{i,k}\, x_{p_j}^k + x_{p_j}^n \left(- \sum_{k=N_0+1}^{n-1} w_{i.j} \frac{x_{p_j}^k}{x_{p_j}^n}\right)$, by (10).

$= x_{p_j}^i \;, \; \forall\, i = 1, \dots, N_0.$

b) $\sum_{k=1}^{n} w_{i,k}\, x_{p_j}^k = \sum_{k=1}^{N_0} w_{i,k}\, x_{p_j}^k + \sum_{k=N_0+1}^{n} w_{i,k}\, x_{p_j}^k$

$$= x_{p_j}^i w_{i,,i} + \sum_{\substack{k=N_0+1\\ k\neq i}}^{n} w_{i,k}\, x_{p_j}^k$$

$$= x_{p_j}^i \Big(1 - \sum_{\substack{k=N_0+1\\ k\neq i}}^{n} w_{i.k} \frac{x_{p_j}^k}{x_{p_j}^i}\Big) + \sum_{\substack{k=N_0+1\\ k\neq i}}^{n} w_{i,k}\, x_{p_j}^k$$

$$= x_{p_j}^i \;, \; \forall\, i = N_0 + 1, \dots, n\,.$$

Theorem 3. Let W be the matrix given by (21), and $x_{a_j} = - x_{p_j}, \forall\, j = 1, \dots, r$.
Then:

$$W x_{a_j} = x_{a_j}, \;\; \forall\, j = 1, \dots, r. \tag{23}$$

Proof. Since (22): $W x_{p_j} = x_{p_j}, \;\; \forall\, j = 1, \dots, r\,.$

$$-W x_{p_j} = -x_{p_j}$$

$$W(-x_{p_j}) = -x_{p_j}$$

Therefore: $W x_{a_j} = x_{a_j}, \forall\, j = 1, \dots, r$.

4. Construction of Discrete Neural Network

Now, we construct the new polynomial discrete recurrent neural network with more than one attractor fixed point given previously.

Let $x_0 = -1, \; x_1 = 1$ be, and we denote by:

a) $f_{-}(x) = A^{-}x^2 + B^{-}x + C^{-}$, the function given by (3).

b) $f_{+}(x) = A^{+}x^2 + B^{+}x + C^{+}$, the function given by (4).

Let $x_0 = -1\,, \; x_1 = 0\,, \; x_2 = 1$ be, and we denote by:

$f_{\pm}(x) = A^{\pm}x^3 + B^{\pm}x^2 + C^{\pm}x + D^{\pm}$, the function given by (7).

The new polynomial discrete recurrent neural network is given by the application: $F\colon \mathbb{R}^n \to \mathbb{R}^n$ defined by $F(x) = (F_1(x), \dots, F_n(x))$, where:

$$F_i(x) = \begin{cases} f_{-}\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right), & \text{if } z_0^i = -r \\ f_{+}\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right), & \text{if } z_0^i = r\,, \quad \forall i = 1, \dots, n \\ f_{\pm}\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right), & \text{if } z_0^i = 0. \end{cases} \tag{24}$$

Theorem 4. Let $x_{p_1}, x_{p_2}, \dots, x_{p_r} \in H^n$ be given by (15), and $F\colon R^n \to R^n$ given by (24).
Then:

a) $F(x_{p_k}) = x_{p_k}\,, \;\; \forall\, k = 1, \dots, r.$ (25)

b) $F(x_{a_k}) = x_{a_k}\,, \;\; \forall\, k = 1, \dots, r.$ (26)

Proof. Since (22): $W x_{p_k} = x_{p_k}$, $\forall\, k = 1, \dots, r$

a) Let W_i be the i-th row of W matrix, then by (22) we obtain:

$$W_i x_{p_k} = x^i_{p_k} \;,\; \forall\, i = 1, \dots, n$$

With which:

$$F_i(x_{p_k}) = \begin{cases} f_-(\sum_{j=1}^{n} w_{i,j}\, x^j), \text{if } z_0^i = -r \\ f_+(\sum_{j=1}^{n} w_{i,j}\, x^j)\,, \text{if } z_0^i = r\,, \quad \forall i = 1, \dots, n \\ f_\pm\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right), \text{if } z_0^i = 0. \end{cases}$$

$$F_i(x_{p_k}) = \begin{cases} f_-(x^i_{p_k}) = x^i_{p_k} \;,\; \text{if } z_0^i = -r \\ f_+(x^i_{p_k}) = x^i_{p_k}\,, \quad \text{if } z_0^i = r\,, \quad \forall i = 1, \dots, n \\ f_\pm(x^i_{p_k}) = x^i_{p_k} \;, \quad \text{if } z_0^i = 0. \end{cases}$$

$F_i(x_{p_k}) = x^i_{p_k}$, $\forall\, k = 1, \dots, n.$

Therefore: $F(x_{p_k}) = x_{p_k}$, $\forall\, k = 1, \dots, r$

b) This proof is analogous to the case (a).

5. Stability

In this section, we study the stability of the fixed points given a priori, guaranteeing that they will be attractive fixed points.

Let $F: R^n \to R^n$ be given by (24), where:

$$F_i(x) = \begin{cases} A_i^-(\sum_{j=1}^{n} w_{i,j}\, x^j)^2 + B_i^-\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right) + C_i^-, \text{if } z_0^i = -r \\ A_i^+(\sum_{j=1}^{n} w_{i,j}\, x^j)^2 + B_i^+\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right) + C_i^+\,, \text{if } z_0^i = r\,, \quad \forall i = 1, \dots, n \\ A_i^\pm(\sum_{j=1}^{n} w_{i,j}\, x^j)^3 + B_i^\pm(\sum_{j=1}^{n} w_{i,j}\, x^j)^2 + C_i^\pm\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right) + D_i^\pm\,, \text{if } z_0^i = 0 \end{cases}$$

which is differentiable of $C^\infty(\mathbb{R}^n)$ class. The Jacobian matrix of F is:

$$JF(x) = \left(\frac{\partial F_i(x)}{\partial x^k}\right)_{nxn}. \tag{27}$$

$$\frac{\partial F_i(x)}{\partial x^k} = \begin{cases} (2A_i^-\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right) + B_i^-)w_{i,k}, \text{ if } z_0^i = -r & (28) \\ (2A_i^+\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right) + B_i^+)w_{i,k}\,, \quad \text{if } z_0^i = r\,, \quad \forall i = 1,\dots,n & (29) \\ (3A_i^\pm(\sum_{j=1}^{n} w_{i,j}\, x^j)^2 + 2B_i^\pm\left(\sum_{j=1}^{n} w_{i,j}\, x^j\right) + C_i^\pm)w_{i,k}\,, \text{ if } z_0^i = 0\,. & (30) \end{cases}$$

Theorem 5. Let $x_{p_1}, x_{p_2}, \dots, x_{p_r} \in H^n$ be given by (15), and $F: \mathbb{R}^n \to \mathbb{R}^n$ given by (24).
Then:

$$\left\| JF(x_{p_t}) \right\|_\infty < \|W\|_\infty \,, \ \forall t = 1, \dots, r. \tag{31}$$

Proof. By (28):

$$\begin{aligned} \textstyle\sum_{k=1}^{n} \left|\frac{\partial F_i(x_{p_t})}{\partial x^k}\right| &= \textstyle\sum_{k=1}^{n} \left|(2A_i^-\left(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j\right) + B_i^-)w_{i,k}\right| \\ &= \textstyle\sum_{k=1}^{n} \left|(2A_i^-\left(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j\right) + B_i^-)\right| \left|w_{i,k}\right| \\ &= \textstyle\sum_{k=1}^{n} \left|(2A_i^-\left(x_{p_t}^i\right) + B_i^-)\right| \left|w_{i,k}\right| \text{, by using (3):} \\ &\le \textstyle\sum_{k=1}^{n} \left|w_{i,k}\right| \end{aligned} \tag{32}$$

By (29):

$$\begin{aligned} \textstyle\sum_{k=1}^{n} \left|\frac{\partial F_i(x_{p_t})}{\partial x^k}\right| &= \textstyle\sum_{k=1}^{n} \left|(2A_i^+\left(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j\right) + B_i^+)w_{i,k}\right| \\ &= \textstyle\sum_{k=1}^{n} \left|(2A_i^+\left(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j\right) + B_i^+)\right| \left|w_{i,k}\right| \\ &= \textstyle\sum_{k=1}^{n} \left|(2A_i^+\left(x_{p_t}^i\right) + B_i^+)\right| \left|w_{i,k}\right| \text{, by using (4):} \\ &\le \textstyle\sum_{k=1}^{n} \left|w_{i,k}\right|. \end{aligned} \tag{33}$$

By (30):

$$\begin{aligned} \textstyle\sum_{k=1}^{n} \left|\frac{\partial F_i(x_{p_t})}{\partial x^k}\right| &= \textstyle\sum_{k=1}^{n} \left|(3A_i^\pm(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j)^2 + 2B_i^\pm\left(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j\right) + C_i^\pm)w_{i,k}\right| \\ &= \textstyle\sum_{k=1}^{n} \left|3A_i^\pm(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j)^2 + 2B_i^\pm\left(\sum_{j=1}^{n} w_{i,j}\, x_{p_t}^j\right) + C_i^\pm\right| \left|w_{i,k}\right| \\ &= \textstyle\sum_{k=1}^{n} \left|3A_i^\pm(x_{p_t}^i)^2 + 2B_i^\pm\left(x_{p_t}^i\right) + C_i^\pm\right| \left|w_{i,k}\right| \text{, by using (7):} \\ &\le \textstyle\sum_{k=1}^{n} \left|w_{i,k}\right| . \end{aligned} \tag{34}$$

Finally, by (32), (33) y (34), we obtain:

$$\left\| JF(x_{p_t}) \right\|_\infty < \|W\|_\infty \,, \ \forall\, t = 1, \dots, r.$$

6. Application

In this section we give an application of our polynomial discrete recurrent neural network to the recognition of four patterns.

Example. Consider four patterns as shown in the figure 4.

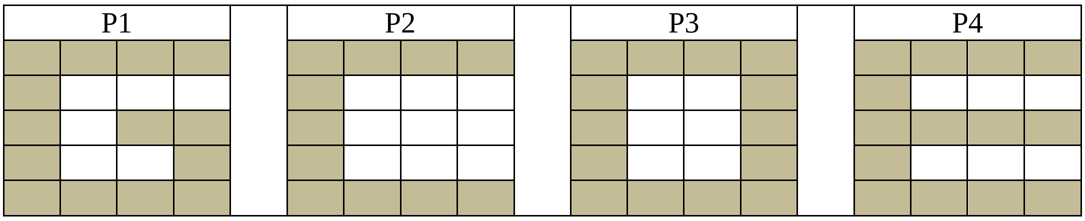
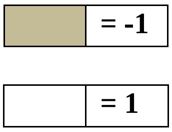


Fig.4. The four patterns as attracting fixed points.

Note that for each pattern we will need 20 neurons. Now, to encode we will use



Coding the patterns we get:

P1	-1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	1	-1	-1	-1	-1	-1

P2	-1	-1	-1	-1	-1	1	1	1	-1	1	1	1	-1	1	1	1	-1	-1	-1	-1

P3	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1

P4	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	-1

Now, disturbing the patterns P1, P2, P3 and P4, and using our polynomial discrete recurrent neural network, as can be seen in the figure 5, our neural network allows to regenerate the patterns.

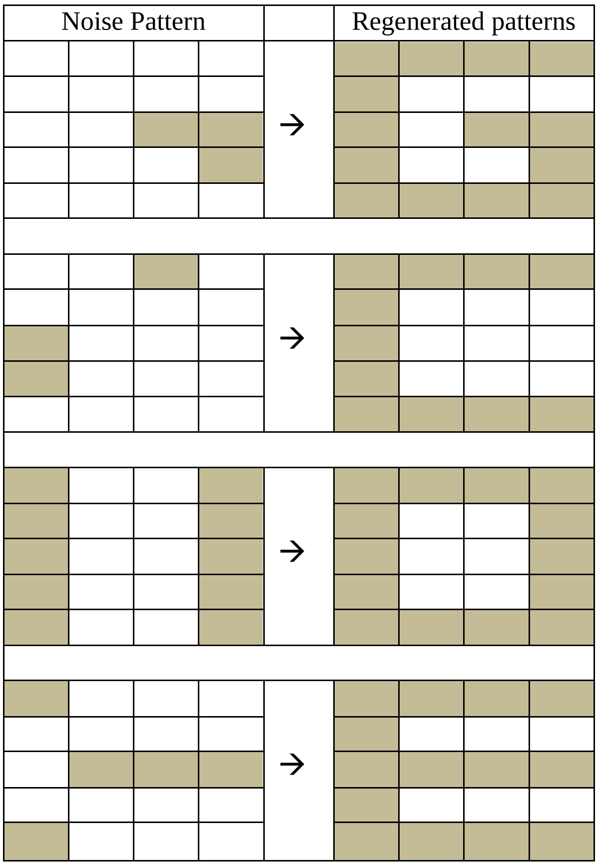


Fig.5. Result of the polynomial discrete recurrent neural network.

7. Conclusion

In this paper we construct a new polynomial discrete recurrent neural network with several fixed points attractors given previously, using the fixed points of quadratic and cubic functions given by (1)-(7).

Using the relations given by (16)-(20); we construct the matrix W of the synaptic weights. In theorem 5 it is proved that the norm of the Jacobian matrix associated with the neural network at fixed points is less than the norm of the matrix W; which guarantees the stability of the fixed points; methodology different from that used by Hopfield [2], which makes use of the energy function associated to the system. This new polynomial discrete recurrent neural network behaves as autoassociative memory; allowing to reconstruct objects from certain information; as in the recognition of images, sounds; as in the application example to the recognition of four patterns.

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